B.Sc Semester VI

Paper - XIII

Model Questions

Group - A [Laplace Transform]

Reference Book - Integral Transform by Goyal & Gupta.

Short Answer Type Questions.

(1) If $L\{F(t)\} = f(s)$

then prove that $L\{F(at)\} = \frac{1}{a} f\left(\frac{s}{a}\right)$.

(2) Find

(a) $L\{t^5 e^{3t}\}$ (b) $L[3e^{3t} + 5t^4 - 4\cos(3t)]$

(3) Find the Laplace transform of

(a) $L(t\cos at)$ (b) $L\{t^2 \sin at\}$.

(4) Find inverse Laplace transform of

(a) $L^{-1}\left\{\frac{s}{s^2+9}\right\}$

(b) $L^{-1}\left\{\frac{1}{s-2} + \frac{2}{s+5} + \frac{6}{s^4}\right\}$.

(5) Find the value of

$L^{-1}\left\{\frac{1}{(s-9)^3}\right\}$.

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Long Answer type Questions.

(1) If $L\{F(t)\} = f(s)$

then prove that $L\{t^n F(t)\} = (-1)^n \frac{d^n}{ds^n} f(s)$

for $n = 1, 2, 3 \cdots$

(2) If $L\{F(t)\} = \{f(s)\}$, then

prove that $L\left\{\frac{F(t)}{t}\right\} = \int_s^\infty f(x)\,dx$

provided the integral exist.

(3) Find the Laplace transform of $\frac{\sin at}{t}$.
Does the transform of $\frac{\cos at}{t}$ exist?

(4) If $L^{-1}\left[\frac{p}{(p^2+1)^2}\right] = \frac{t}{2}\sin t$, then

find $L^{-1}\left[\frac{32p}{(16p^2+1)^2}\right]$

(5) State and prove Convolution Theorem.

or

If $L^{-1}\{f(s)\} = F(t)$, $L^{-1}\{g(s)\} = G(t)$

then prove that

$$L^{-1}\{f(s)\,g(s)\} = \int_0^t F(u)\,G(t-u)\,du$$

$$= F * G$$

$$= \int_0^t F(t-u)\,G(u)\,du$$

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(6) Find the inverse Laplace transform of $\frac{1}{s^2(s^2-a^2)}$ by using the Convolution theorem.

(7) Use the Convolution theorem to find

(i) $L^{-1}\left\{\frac{p^2}{(p^2+4)^2}\right\}$

(8) Using Laplace transform method, Solve $y''(t) + y(t) = t$ given that $y'(0) = 1$, $y(\pi) = 0$.

(9) Using Laplace transform, Solve $y'' + y = \cos x$, where $y(0) = 0 = y'(0$

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MODEL QUESTION FOR END SEMESTER

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Group-B (operation Research)

Short-type question.

1) Defin convex set.
Prove that the intersection of any finite number of convex sets is a convex set

2) Defin convex combination.
Prove that the set of all convex combinations of a finite number of points $x_1, x_2, \ldots, x_n$ is a convex set.

3) Prove that dual of the dual of a given primal is the primal itself.

4) Write the dual of the following LPP

$$\max. Z = 2x_1 + 3x_2 + x_3$$

S.t.

$$4x_1 + 3x_2 + x_3 = 6$$
$$x_1 + 2x_2 + 5x_3 = 4$$

and $x_1, x_2, x_3 \geq 0$

5) Write the difference between Primal and dual problems.

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Long-type Question

1). Solve the following problem by Simplex Method.

$$\max. Z = 2x_1 + 4x_2$$

S.t.

$$2x_1 + 3x_2 \leq 48$$
$$x_1 + 3x_2 \leq 42$$
$$x_1 + x_2 \leq 21$$

and $x_1, x_2 \geq 0$

2) Solve the following LPP by Big-M method

$$\min Z = 2x_1 + x_2$$

S.t.

$$3x_1 + x_2 = 3$$
$$4x_1 + 3x_2 \geq 6$$
$$x_1 + 2x_2 \leq 3$$

and $x_1, x_2 \geq 0$

3) Solve Graphically the following LPP.

$$\min. Z = 2x_1 + 3x_2$$

S.t.

$$x_1 + x_2 \leq 4$$
$$6x_1 + 2x_2 \geq 8$$
$$x_1 + 5x_2 \geq 4$$
$$x_1 \leq 3$$
$$x_2 \leq 3$$

and $x_1, x_2 \geq 0$

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4) Solve the following problem by Simplex Method

$$\max Z = 3x_1 + 4x_2$$

s.t.

$$x_1 + 3x_2 \leq 9$$
$$2x_1 - x_2 \leq 8$$
$$x_1 + x_2 \leq 5$$

and $x_1, x_2 \geq 0$

5) Solve the assignment problem represented by the following matrix.

	I	II	III	IV	V	VI
A	9	22	58	11	19	27
B	43	78	72	50	63	48
C	41	28	91	37	45	33
D	74	42	27	49	39	32
E	36	11	57	22	25	18
F	3	56	53	31	17	28

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