MODEL QUESTIONS

For Mathematics (GE-2)

Semester 2

Short Answer Questions

1. If u= log $(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = \frac{-9}{(x+y+z)^2}$
2. If $= sin^{-1} \frac{\sqrt{x}-\sqrt{y}}{\sqrt{x}+\sqrt{y}}$, Show that $\frac{\partial u}{\partial x} = -\frac{y}{x}\frac{\partial u}{\partial y}$.
3. If u= $\sin^{-1}\frac{x}{y} + \tan^{-1}\frac{y}{x}$, Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 0$
4. Find the radius of curvature at any point of the parabola $y^2 = 4ax$ and deduce that the radius of curvature at parabola is equal to semi-latus rectum.
5. Find the radius of curvature for the curve, s= c tan ϕ at point (s, ϕ).
6. Prove that $\int_0^{\frac{\pi}{2}} \frac{sin2\theta}{sin^4\theta+cos^4\theta} d\theta = \frac{\pi}{2}$.
7. Find the radius of curvature at any point for the curve, $r = ae^{m\theta}$
8. Calculate Divergence of the vector field:

$$\vec{f} = \frac{x}{x+y+z}\vec{i} + \frac{y}{x+y+z}\vec{j} + \frac{z}{x+y+z}\vec{k}$$

9. Evaluate $\int_0^a \frac{x^2}{(a^2-x^2)^{1/2}} dx$
10. Evaluate $\int_0^{\frac{\pi}{2}} \frac{\sqrt{cosx}}{\sqrt{cosx}+\sqrt{sinx}} dx$
11. Evaluate $\lim_{n\to\infty} \left\{\left(\frac{n}{n^2+1^2}\right)\left(\frac{n}{n^2+2^2}\right) + \cdots + \left(\frac{n}{2n^2}\right)\right\}$
12. Evaluate $\int_0^{\frac{\pi}{4}} tan^5 x dx$ using reduction formula
13. Calculate the divergence of the vector field

$$\vec{f} = \frac{z-x}{x^2+y^2+z^2}\vec{i} + \frac{x-y}{x^2+y^2+z^2}\vec{j} + \frac{y-z}{x^2+y^2+z^2}\vec{k}$$

14. If $\vec{r} = x\vec{i} + y\vec{j} + z\vec{k}$ and $\vec{a}$ is a constant, prove that, Div. $[\vec{a} \times (\vec{r} \times \vec{a})] = 2a^2$

(GE-2)

Long Answer Questions

1. If u is a homogeneous function of nth order in x, y, z such that u = $x^n f\left(\frac{y}{x}, \frac{z}{x}\right)$, then Prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = nu$
2. If ρ be the radius of curvature of a parabola at a point whose distance along curve from fixed point is *s* then $3\rho\frac{\partial^2 \rho}{\partial s^2} - \left(\frac{\partial \rho}{\partial s}\right)^2 - 9 = 0.$
3. If u be a homogeneous function of nth degree in x, y, z and if u =f (X,Y,Z) where X,Y,Z are differential coefficients of 'u', w.r.t. x, y, z respectively then Prove that, $X\frac{\partial f}{\partial X} + Y\frac{\partial f}{\partial Y} + Z\frac{\partial f}{\partial Z} = \frac{n}{n-1}u$
4. Write the working rule to find the asymptotes on the algebraic curve and find real asymptotes of the curve $x^3 + y^3 = 3axy$.
5. Obtain the polar tangential formula for radius of curvature.
6. If r=a sec 2θ, Prove that $\rho = \frac{r^4}{3p^3}$
7. Prove that $\int_0^{\frac{\pi}{2}} \cos^{n-2}x \sin nx dx = \frac{1}{n-1}$
8. Evaluate $\int_\alpha^\beta \sqrt{(x-\alpha)(\beta - x)}\, dx$
9. Evaluate
   a. $\int_0^{\frac{\pi}{2}} \frac{\cos x}{(1+\sin x)+(1+\cos x)} dx.$
   b. $\int_0^{\frac{\pi}{2}} \frac{x^2}{(x\sin x+\cos x)^2} dx.$
10. Evaluate $\lim_{n\to\infty} \left\{\left(1+\frac{1}{n^2}\right)\left(1+\frac{4}{n^2}\right)\left(1+\frac{9}{n^2}\right)\ldots\ldots(2)\right\}^{1/n}$
11. Evaluate $\int_0^{\frac{\pi}{2}} \sin^n x dx$
12. Evaluate $\int_0^{\frac{\pi}{2}} \sin^m x \cos^n x dx$ such that m, n>0 and m, n ϵ I
13. Prove that grad $\left(\frac{f}{g}\right) = \frac{g\,(grad\, f) - f\,(grad\, g)}{g^2}$
14. Find the derivative of *f* at *P* in the direction of $\vec{a}$ where $f = e^{yt}\cos x + e^{zx}\cos y + e^{xy}\cos z$ $P\left(\frac{\pi}{6}, \frac{\pi}{3}, 0\right)$, $\vec{a} = 3\vec{i} + 2\vec{j} - \vec{k}$
15. Prove that $\nabla \times (\phi \vec{f}) = \phi(\nabla \times \vec{f}) + (\nabla\phi) \times \vec{f}$
16. Calculate Curl $\vec{f} = (x^2+y^2+z^2)^{-1/2}(yz\,\vec{i} + zx\,\vec{j} + xy\,\vec{k})$